① $\int_0^1\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}}\,dy\,dx = \int_0^1\int_{x^2+1}^{x^2+2} x\,\frac{1}{\sqrt{u}}\,\frac{du}{2}\,dx = \int_0^1 x\sqrt{u}\Big]_{x^2+1}^{x^2+2} dx = \int_0^1 \left(x\sqrt{x^2+2} - x\sqrt{x^2+1}\right)dx =$

$u = x^2+2$ ⋮ $\quad u = x^2+1$ ⋮ $\quad \frac{1}{3} - \frac{4\sqrt{3}}{3} + \sqrt{5}$

$u = x^2+y^2+1$

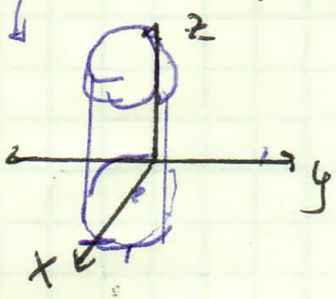
$du = 2y\,dy$

② surface $x^2+y^2-x=0$

$x^2 - x + \frac{1}{4} + y^2 = 0 + \frac{1}{4}$

$(x-1/2)^2 + y^2 = \frac{1}{4}$

Cylinder centered at $(1/2, 0)$



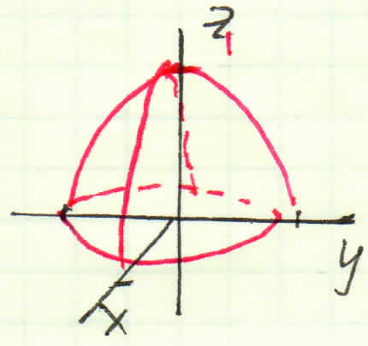
Convert to Cylindrical

$x^2+y^2-x=0$

$r^2 - r\cos\theta = 0$

$r = \cos\theta$

surface $z = 1-x^2-y^2$

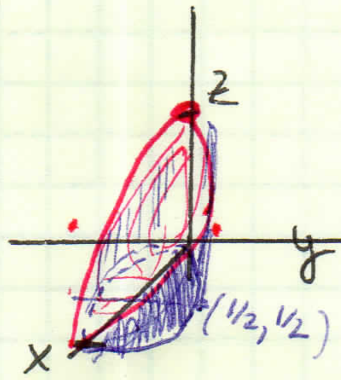


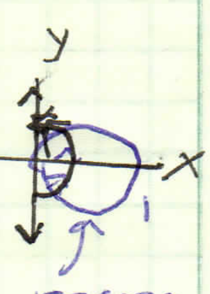
Convert

$z = 1-(x^2+y^2)$

$z = 1-r^2$

Solid





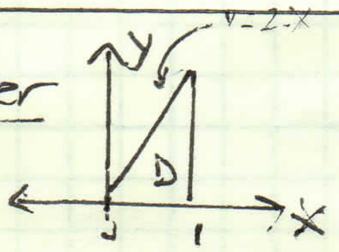
$r = \cos\theta$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$\int_{-\pi/2}^{\pi/2}\int_0^{\cos\theta} (1-r^2)\,r\,dr\,d\theta$$

③ Reverse Order

can't integrate $\cos(x^2)$ easily



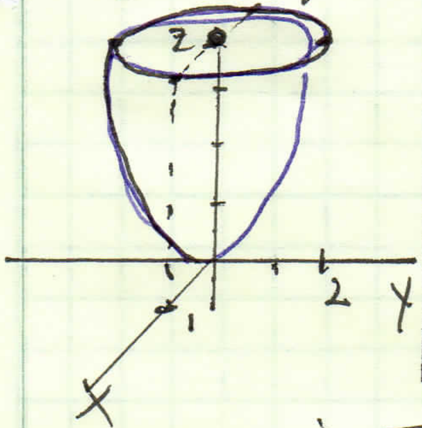
$\int_0^1\int_0^{2x} \cos(x^2)\,dy\,dx = \int_0^1 y\cos(x^2)\Big]_0^{2x} dx$

$= \int_0^1 2x\cos(x^2)\,dx = \sin(x^2)\Big]_0^1 = \sin 1$

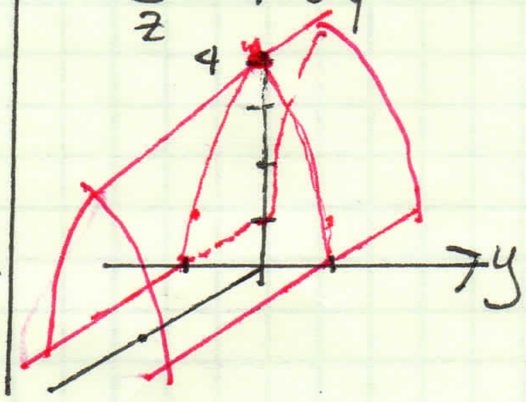
$u = x^2$

④ ✓

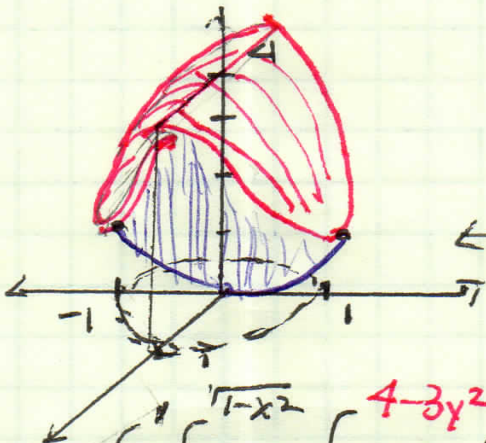
⑤ $z = 4x^2+y^2$

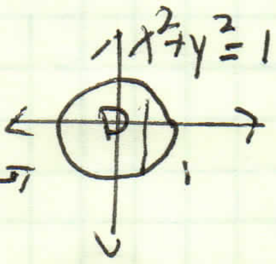


$z = 4-3y^2$



Solid





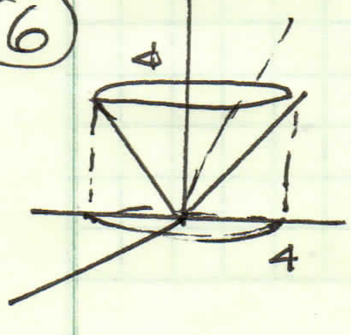
intersection $4x^2+y^2 = 4-3y^2$

$4y^2 + 4y^2 = 4$

$x^2+y^2=1$

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{4x^2+y^2}^{4-3y^2} f(x,y,z)\,dz\,dy\,dx$$

⑥



(b) Convert $z=4$ to spherical

$z = 4$

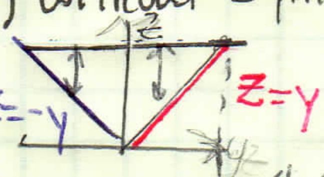
$\rho\cos\phi = 4$

$\rho = \frac{4}{\cos\phi} = \sec\phi$

c) without symmetry

$$\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \left(4-\sqrt{x^2+y^2}\right)dy\,dx$$

d) without symmetry



$$\int_{-4}^{0}\int_{-y}^{4}\int_{-\sqrt{z^2-y^2}}^{\sqrt{z^2-y^2}} dx\,dz\,dy + \int_0^4\int_y^4\int_{-\sqrt{z^2-y^2}}^{\sqrt{z^2-y^2}} dx\,dz\,dy$$

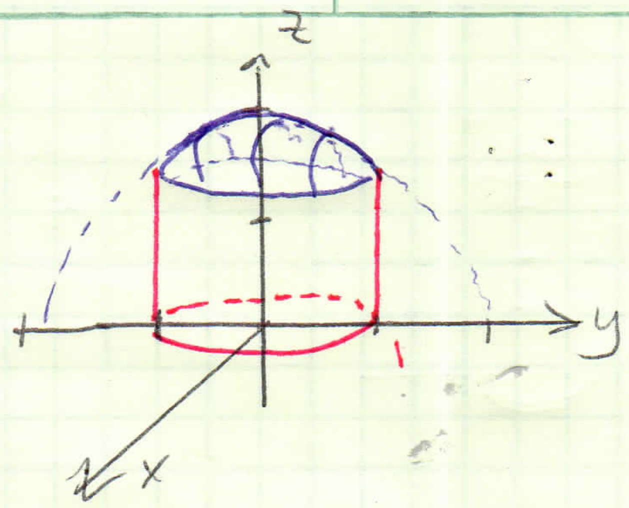
⑦ Recreate solid

$0 \le z \le \sqrt{4-r^2}$

$0 \le z \le \sqrt{4-x^2-y^2}$ ← hemispher

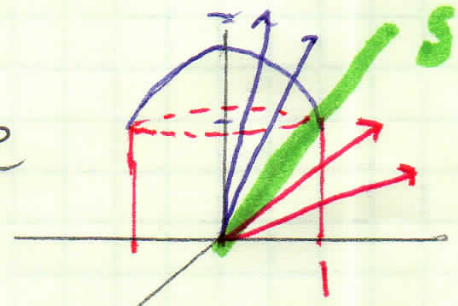
$0 \le r \le 1$

$0 \le \theta \le 2\pi$



b) $$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} dz\,dy\,dx$$

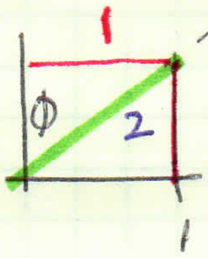
c) ρ goes to sphere then change to cylinder.



split

Find ϕ at split:

either convert: OR trig



$\Rightarrow \sin\phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{6}$ at split.

Intersection

$\begin{cases} x^2+y^2+z^2=4 \\ x^2+y^2=1 \end{cases}$

$\begin{cases} \rho=2 \\ r=1 \Rightarrow \rho\sin\phi=1 \end{cases}$

$\Rightarrow 2\sin\phi = 1 \Rightarrow \phi = \pi/6$

Convert cylinder to spherical

$x^2+y^2=1$

$r^2=1$

$(\rho\sin\phi)^2=1$

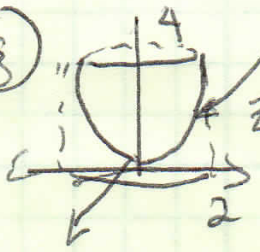
$\rho\sin\phi=1$

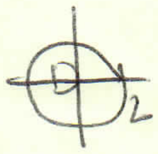
$\rho=\frac{1}{\sin\phi}=\csc\phi$

out sphere / out cylinder

$$\int_0^{2\pi}\int_0^{\pi/6}\int_0^{2}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta + \int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^{\csc\phi}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

⑧





$g(x,y)$ $z=x^2+y^2$

$dS=\sqrt{4x^2+4y^2+1}\,dA$

$$\iint_S z\,dS = \iint_D (x^2+y^2)\sqrt{4x^2+4}$$

⑨ omit

$$= \int_0^{2\pi}\int_0^{2} r^2\sqrt{4r^2+1}\; r\,dr\,d\theta \ldots$$

$u=4r^2+1$ ...

⑩ $ds=\sqrt{1+4\cos^2t+4\sin^2t}\,dt = \sqrt{5}\,dt$

$$\int_C yz\cos x\,ds = \int_0^{\pi} 2\cos t\, 2\sin t\, \cos t\,\sqrt{5}\,dt \ldots$$

$u=\cos t$ ...